International Journal of Theoretical Physics, Vol. 36, No. 12, 1997

Kinematics of Edge Dislocations. II. Orowan-Type Kinematic Relations

A. Trzęsowski[1]

Received May 9, 1997

The kinematics of edge dislocations based on the model of dislocation lines located in a time-dependent equidistant Riemannian material space is considered. The distinguished material flows, called dislocation flows, are introduced in order to make a comparison between the dislocation kinematics and the macroplasticity kinematics. It is shown that the Riemannian generalization of kinematic assumptions of the perfect plasticity theory leads to Orowan-type formulas for the glide as well as for the double cross-slip types of the edge dislocation kinematics. The influence of driving stress of moving dislocations on the shear rate is discussed.

1. INTRODUCTION

There are various elementary acts of plasticity due to the dislocation motion. Their recognition is essentially the subject of the physical theory of plasticity (microplasticity) (Trzęsowski, 1998, Section 1). The mesoplastic approach (Trzęsowski, 1998, Section 1), the subject of this paper, is based on the application of microplasticity concepts on a mesoscale level, where the concept of a continuized crystal (e.g., Kröner, 1986; Trzęsowski, 1993) is still applicable. Thus, let us start with a list of some dislocation mechanisms leading to the occurrence of plastic deformations.

It is known (e.g., Hull and Bacon, 1984) that the *glide motion* of a dislocation, in which it moves in the surface containing its line and Burgers vector, is an elementary act of plasticity. The glide motion of many dislocations results in *slip*, which is the most common manifestation of plastic deformation in crystalline solids. It can be envisaged as (local) sliding or successive displacements of one plane of atoms over another on the so-called

[1] Institute of Fundamental Technological Research, Polish Academy of Sciences 00-049 Warsaw, Poland.

0020-7748/97/1200-2895$12.50/0

(local) *slip planes.* Consequently, a *dislocation line* can be considered, in the continuous description, as the one defining a boundary between slipped and unslipped parts of the crystal (Hull and Bacon, 1984). Moreover, globally (i.e., on the macroscale) the occurrence of *slip surfaces* is observed, because the appearance of dislocation lines generates a bend in originally straight lattice lines. For example, in the case of the so-called *single glide* (in which the crystal deforms by slip on one set of parallel crystal planes only) lattice lines originally normal to the plane of slip form a normal congruence, i.e., the curves of the congruence are orthogonal trajectories of a family of (virtual) slip surfaces (Bilby *et al.*, 1958). The slip surfaces are called then also (single) *glide surfaces.*

Another example of surfaces produced by local slips occurs in the case of the so-called *cross-slip motion* (Hull and Bacon, 1984). Namely, discrete blocks of a crystal between two slip planes remain undistorted, and further deformation occurs either by more movement on existing slip planes or by formation of new slip planes. In this last case plastic deformation may occur in the form of *slip bands.* Each band is made up of a large number of slip steps on closely spaced slip planes. In this case moving dislocations do not lie on the same glide plane, but on a set of parallel glide planes, and they switch from one glide plane to another. This process is called cross-slip. For example, in the case of *double cross-slip*, the Burgers vector is parallel to a slip plane, but the dislocation line is bent in such a way that one part lies on the slip plane and the other on the plane parallel to it. The cross-slip produces a nonplanar slip surface.

It is shown in this paper that a model of the kinematics of edge dislocations (Section 3) based on the geometric theory of a continuous distribution of dislocations (Section 2) enables us to distinguish, in terms of an intrinsic rate of stretchings tensor (Section 3), the elementary act of plasticity of single glide character from that of double cross-slip character (Section 4). The Orowan-type kinematic relations (Trzęsowski, 1998, Section 1) corresponding to these elementary acts of plasticity are derived (Section 4), and the influence of driving stress of moving dislocations on the shear rate is discussed (Section 5).

2. GLIDE SURFACES

Let $\Phi(t) = (\mathbf{E}_a(\cdot, t);\ a = 1, 2, 3)$ and $\Phi^*(t) = (E^a(\cdot, t))$, $t \in I \subset R_+$ denote the (time-dependent) Bravais moving frame and the Bravais moving coframe dual to it, respectively (Trzęsowski, 1998):

$$\mathbf{E}_a = \underset{a}{e}^A \partial_A, \qquad E^a = \overset{a}{e}_A\, dX^A$$

$$\langle E^a, \mathbf{E}_b \rangle = \overset{a}{e}_A\, \underset{b}{e}^A = \delta^a_b, \qquad [E^a] = \text{cm}, \qquad [\mathbf{E}_a] = \text{cm}^{-1} \qquad (2.1)$$

where a dimensional Lagrange coordinate system $X = (X^A;\ A = 1, 2, 3)$, $[X^A] = \mathrm{cm}$, is considered. The *intrinsic metric* tensor $\mathbf{g}$ defined by Φ has the form

$$\mathbf{g}(X, t) = \delta_{ab}E^a(X, t) \otimes E^b(X, t) = g_{AB}(X, t)\, dX^A \otimes dX^B$$

$$g_{AB}(X, t) = \overset{a}{e}_A(X, t)\, \overset{b}{e}_B(X, t)\delta_{ab}, \qquad [\mathbf{g}] = \mathrm{cm}^2 \tag{2.2}$$

and defines a time-dependent material Riemannian space $\mathcal{B}_g = (\mathcal{B}, \mathbf{g})$, where $\mathcal{B}$ denotes the body identified with an open, simply connected subset of the Euclidean point space E^3 (the configurational space of the body). The *instantaneous material space* will be denoted by $\mathcal{B}_t = (\mathcal{B}, \mathbf{g}_t)$, where $\mathbf{g}_t(X) = \mathbf{g}(X, t)$. The intrinsic metric tensor represents the existence of secondary point defects created by the distribution of dislocations (defined by Φ) in a way consistent with the property that dislocations have no influence on local metric properties of a crystal structure (Trzęsowski, 1998). The influence of these secondary point defects on the dislocation kinematics will be modeled by treating dislocation lines (see Section 1 and Trzęsowski, 1998) as those located in the material Riemannian space $\mathcal{B}_g$ (Trzęsowski, 1998).

Let us assume the existence of a *convective Lagrange coordinate system* $X = (X^A) = (X^\kappa, X^3)$ on $\mathcal{B}$ (Trzęsowski, 1998):

$$X^A = \chi^A(\xi, t), \qquad \chi^A(\xi, 0) = \delta^A_a\xi^a \tag{2.3}$$

where $\xi = (\xi^a)$ denotes a reference Lagrange coordinate system, such that the time-dependent intrinsic metric tensor takes the following form in these coordinates:

$$\mathbf{g}(X, t) = \mathbf{g}_t(X) \doteq \Psi_t(X^3)\mathbf{a}_t(X^\kappa) + dX^3 \otimes dX^3 \tag{2.4}$$

where $\mathbf{a}_t$ is the metric tensor (depending on $t \in I$ as a parameter) of a general two-dimensional Riemannian space, and Ψ_t, $t \in I$, is a positive non-dimensional scalar, say of the form

$$\Psi_t(X^3) = a(t)^2 \exp[-2\kappa_t(X^3)], \qquad \Psi_t(0) = 1 \tag{2.5}$$

It is the canonical form of a metric tensor of the so-called *equidistant Riemannian space* $\mathcal{B}_g$ (Trzęsowski, 1998). Moreover, in this case, there exists a distinguished $\mathbf{g}$-unit vector field $\mathbf{n}$ such that (in the convective coordinates)

$$\mathbf{n} = n^A\partial_A, \qquad n^A \doteq \delta^A_3 \tag{2.6}$$

and the curves in $\mathcal{B}_g$ tangent to $\mathbf{n}$ form a geodesic congruence of curves orthogonal to the coordinate surfaces (of the convective coordinate system) defined, at least locally, as

$$\Sigma_c = \{p \in U: X^3(p) = c\} \tag{2.7}$$

where $c \in R$, $[c] = \text{cm}$, are constants and $U \subset \mathcal{B}_g$ is a coordinate neighborhood (Trzęsowski, 1998). The convective Lagrange coordinate system defines *actual* (Lagrange) coordinates on $\mathcal{B}_g$. The intrinsic metric tensor (2.4) corresponds, e.g., to a Bravais moving frame defined in actual coordinates by (Trzęsowski, 1998)

$$\mathbf{E}_\alpha(X, t) \doteq \Psi_t^{-1/2}(X^3)\mathbf{e}_\alpha(X^\kappa, t) \tag{2.8a}$$

$$\mathbf{E}_3(X, t) \doteq \partial_3, \qquad \alpha, \kappa = 1, 2 \tag{2.8b}$$

and the following time-dependent coordinate transformation:

$$X^\alpha = \chi^\alpha(\xi^\kappa, t), \qquad \chi^\alpha(\xi^\kappa, 0) = \xi^\alpha$$
$$X^3 = \xi^3, \qquad \alpha, \kappa = 1, 2 \tag{2.9}$$

defines a convective Lagrange coordinate system in which the Bravais moving frame has the form (2.8) in the actual coordinates $X = (X^A)$ as well as in the reference coordinate system $\xi = (\xi^a)$.

The coordinate surfaces (2.7) are slices of maximal integral manifolds of a two-dimensional involutive distribution of local slip planes containing base vectors $\mathbf{E}_a$, $\alpha = 1, 2$, of the form (2.8a) everywhere (Trzęsowski, 1998). Namely, the lines in $\mathcal{B}_g$ defined by the condition

$$l_3 = \mathbf{l} \cdot \mathbf{E}_3 = 0 \tag{2.10}$$

where $\mathbf{l}$ is the unit tangent to the line and $\mathbf{u} \cdot \mathbf{v} = \mathbf{ugv}$ are edge dislocation lines for the distribution of dislocations defined by (2.8), and their local Burgers vectors $\mathbf{b}$ are given by

$$\mathbf{b} = b_g\mathbf{m}, \qquad b_3 = \mathbf{b} \cdot \mathbf{E}_3 = 0$$
$$m^a m_a = 1, \qquad \mathbf{m} \cdot \mathbf{l} = 0 \tag{2.11}$$

and

$$\rho b_g = \tfrac{1}{2} |\Psi_t'/\Psi_t| = |\kappa_t'|$$
$$b_g = \|\mathbf{b}\|_g = (b_a b^a)^{1/2} \tag{2.12}$$

where $f' = \partial_3 f$, and $\rho > 0$ is the (volume) scalar density of dislocations. Thus, we can define a *local glide system* ($\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$) such that the above-mentioned maximal integral manifolds are (virtual) *glide surfaces* for edge dislocation lines described by this glide system (Trzęsowski, 1998). These glide surfaces and their slices (2.7) will be denoted by Σ_c. For each $t \in I$, $\Sigma_c \subset \mathcal{B}_t$ and the first fundamental form $\mathbf{a}_{c,t}$ induced on Σ_c from $\mathcal{B}_t$ has, according to (2.4), the following form:

$$\mathbf{a}_{c,t}(X^\kappa) = \Psi_t(c)\mathbf{a}_t(X^\kappa) = a[c, t]_{\alpha\beta}(X^\kappa)\, dX^\alpha \otimes dX^\beta \tag{2.13}$$

where [see (2.8a)]

$$\mathbf{a}_t(X^\kappa) = \delta_{\alpha\beta}e^\alpha(X^\kappa, t) \otimes e^\beta(X^\kappa, t) = a_{\alpha\beta}(X^\kappa, t)\, dX^\alpha \otimes dX^\beta \qquad (2.14a)$$

$$a[c, t]_{\alpha\beta}(X^\kappa) = \Psi_t(c)a_{\alpha\beta}(X^\kappa, t), \qquad \Psi_t(0) = 1 \qquad (2.14b)$$

The submanifolds $\Sigma_{c,t} = (\Sigma_c, \mathbf{a}_{c,t})$ of $\mathcal{B}_t$ are time-dependent umbilical surfaces with the mean curvature $H_t(c)$ given by (Trzęsowski, 1998)

$$H_t(c) = 2\kappa_t'(c)$$

$$\kappa_t' = \partial_3\kappa_t, \qquad [H_t(c)] = \text{cm}^{-1} \qquad (2.15)$$

where the scalar κ_t is that of (2.5). Consequently, it follows from (2.12) and (2.15) that, for the distribution of dislocations defined by (2.8), the formula

$$\rho b_{g|\Sigma_{c,t}} = \tfrac{1}{2}\,|H_t(c)| \qquad (2.16)$$

describing the influence of edge dislocations on the mean curvature of glide surfaces is valid (Trzęsowski, 1998). Note that, according to (2.16), the modulus b_g of the local Burgers vector of a gliding dislocation line is independent of the choice of this line (Trzęsowski, 1998).

3. DISLOCATION KINEMATICS

The partial derivative ∂_t with respect to the time parameter will be designated also (for simplicity) by the dot over letters, e.g. [see (2.1)]

$$\dot{\mathbf{E}}_a(X, t) = \partial_t\, \underset{a}{e}^A(X, t)\partial_A \qquad (3.1)$$

and for the volume 3-form V of the Riemannian material space $\mathcal{B}_g$, we have [see Trzęsowski, 1998, Section 1, and (2.7) and (2.28)]

$$\dot{V} = \dot{\Gamma}V$$

$$\dot{\Gamma} = \dot{e}/e, \qquad \dot{e} = \partial_t e, \qquad \Gamma = \ln e$$

$$e = g^{1/2}, \qquad e = \det(\overset{a}{e}_A), \qquad g = \det(g_{AB}) \qquad (3.2)$$

If $(\mathbf{C}_a;\, a = 1, 2, 3)$ is a Cartesian base on the ambient Euclidean point space E^3 of the body, then the tensor field $\mathbf{P} = \mathbf{P}\,(\cdot, t)$ defined by

$$\mathbf{E}_a(X, t) = \mathbf{P}(X, t)\mathbf{C}_a$$

$$\mathbf{C}_a\boldsymbol{\delta}\mathbf{C}_b = \delta_{ab}, \qquad \mathbf{E}_a(X, t)\mathbf{g}(X, t)\mathbf{E}_b(X, t) = \delta_{ab} \qquad (3.3)$$

where $\boldsymbol{\delta}$ denotes the Euclidean metric on the point space E^3, is called a *plastic distortion* (of the body). It follows from (3.3) that

$$\dot{\mathbf{E}}_a = \mathbf{S}_p\mathbf{E}_a, \qquad \mathbf{S}_p = \dot{\mathbf{P}}\mathbf{P}^{-1} \qquad (3.4)$$

and

$$\dot{\mathbf{g}} = -2\mathbf{D}_p \tag{3.5a}$$

$$\mathbf{D}_p = \tfrac{1}{2}(\mathbf{L}_p + \mathbf{L}_p^T), \qquad \mathbf{L}_p = \mathbf{g}\mathbf{S}_p \tag{3.5b}$$

From (3.2) and (3.5) we obtain

$$\mathrm{tr}\mathbf{D}_p = \dot{J}_p/J_p = -\dot{\Gamma}, \qquad J_p = \det \mathbf{P} = g^{-1/2} \tag{3.6}$$

Therefore, the symmetric tensor field $\mathbf{D}_p$ defines the rate of the change of intrinsic metric properties of the continuized dislocated Bravais crystal and its trace is a measure of the rate of plastic distortion of the Euclidean volume of the body.

Let us consider a *material flow* in the form of a smooth mapping $\chi:\mathcal{B}_g \times I \to \mathcal{B}_g$ such that for each $t \in I$, $\chi_t(\cdot) = \chi(\cdot, t)$ is a local diffeomorphism χ_t: $\mathcal{B}_0 \to \mathcal{B}_t$. If $\xi = (\xi^a)$ is a coordinate system on $\mathcal{B}_0$ and $X = (X^A)$ is a coordinate system on $\mathcal{B}_g$, then we can define a convective Lagrange coordinate system $X^A = \chi^A(\xi, t)$ on $\mathcal{B}$ assuming that for each $p \in \mathcal{B}_0$

$$\chi^A(\xi(p), t) = X^A(\chi(p, t)) \tag{3.7}$$

Conversely, a Lagrange convective coordinate system on $\mathcal{B}$ defines, according to (3.7), a material flow χ: $\mathcal{B}_g \times I \to \mathcal{B}_g$. For example, the coordinate transformation (2.9) defines a material flow preserving the canonical form (2.4) of the intrinsic "equidistant" metric tensor as well as the form (2.8) of the Bravais moving frame. The mapping χ defines a *material velocity* field $\mathbf{v}$ by

$$\mathbf{v}(X, t) = \mathbf{v}_t(X), \qquad \mathbf{v}_t = \mathbf{V}_t \circ \chi_t^{-1}$$
$$\mathbf{V}_t(\xi) = \dot{\varphi}_\xi(t), \qquad \varphi_\xi(t) = \chi(\xi, t) \tag{3.8}$$

where $\dot{\varphi}_\xi$ denotes the vector field tangent to the curve φ_ξ: $I \to \mathcal{B}$, and the mapping χ is identified with its coordinate description (χ^A). In this coordinate description

$$\mathbf{v}(X, t) = v^A(X, t)\partial_A$$
$$v^A(\chi(\xi, t), t) = \partial_t \chi^A(\xi, t), \qquad [v^A] = \text{cm sec}^{-1} \tag{3.9}$$

The mapping χ and the velocity field $\mathbf{v}$ will be called a *dislocation flow* and a *dislocation flow velocity*, respectively. We will take only advantage of the existence of a dislocation flow velocity defined by (3.9) and a Lagrange convective coordinate system. If $(\mathbf{l}, \mathbf{m}, \mathbf{n})$ is a local glide system (Trzęsowski, 1998), then the *glide* component $v_{(m)}$ and the *climb* component $v_{(n)}$ of the dislocation flow velocity are defined by

$$v_{(m)} = \mathbf{v} \cdot \mathbf{m}, \qquad v_{(n)} = \mathbf{v} \cdot \mathbf{n} \tag{3.10}$$

where $\mathbf{u} \cdot \mathbf{v} = \mathbf{ugv}$. For example, if the dislocation flow density is defined by (3.9) and

$$X^\alpha = \chi^\alpha(\xi^a, t), \qquad \chi^\alpha(\xi^a, 0) = \xi^\alpha$$
$$X^3 = \xi^3, \qquad a = 1, 2, 3, \quad \alpha = 1, 2 \tag{3.11}$$

then

$$\mathbf{v}(X, t) = v^\alpha(X, t)\partial_\alpha, \qquad \alpha = 1, 2$$
$$v_{(n)} = v_3 = 0 \tag{3.12}$$

Note that the condition $v_{(n)} = 0$ may be accepted, e.g., at low temperatures (Ghoniem and Amadeo, 1990). The way in which dislocation flows would be associated with a deformation process of the considered material body due to its external loading (and, e.g., due to the influence of a thermal field) is beyond the scope of the paper.

Let us consider, in order to make a comparison between the dislocation kinematics and the macroplasticity kinematics, the following intrinsic counterpart of the spatial velocity gradient:

$$\mathbf{L}_g = \mathbf{g}\nabla^g\mathbf{v} = \nabla^g v, \qquad \text{i.e.,} \quad L_{AB} = \nabla^g_A v_B$$
$$v = \mathbf{gv} = v_A\, dX^A, \qquad v_A = g_{AB}v^B \tag{3.13}$$

where

$$\nabla^g v = \Gamma_A(v) \otimes dX^A$$
$$\Gamma_A(v) = dv_A - v_B\omega^B_A[\mathbf{g}], \qquad \omega^B_A[\mathbf{g}] = \Gamma^B_{AC}[\mathbf{g}]\, dX^C \tag{3.14}$$

and $\nabla^g = (\Gamma^A_{BC}[\mathbf{g}])$ denotes the Levi-Civita covariant derivative corresponding to the intrinsic metric tensor $\mathbf{g}$. If the *intrinsic rate of stretchings tensor* $\mathbf{D}_g$ is defined by

$$\mathbf{D}_g = \tfrac{1}{2}(\mathbf{L}_g + \mathbf{L}_g^T) \tag{3.15}$$

then

$$\mathrm{tr}\mathbf{D}_g = \mathrm{tr}\mathbf{L}_g = g^{AB}L_{AB} = \mathrm{div}_g\mathbf{v} \tag{3.16}$$

where div_g denotes the divergence operator defined by ∇^g:

$$\mathrm{div}_g\mathbf{v} = \nabla^g_A v^A = g^{-1/2}\partial_A(g^{1/2}v^A) \tag{3.17}$$

and g is defined in (3.2). The pullback $\mathbf{g}$ by χ defines an intrinsic counterpart $\mathbf{G}$ of the right Cauchy–Green tensor:

$$\mathbf{G} = \chi^*\mathbf{g} = \delta_{ab}G^a \otimes G^b = G_{ab}\, d\xi^a \otimes d\xi^b$$

$$G_{ab} = X_a^A X_b^B g_{AB} \circ \chi = B_a^c B_b^d \delta_{cd}$$

$$G^a = \chi^* E^a = B_b^a\, d\xi^b, \qquad B_b^a = \overset{a}{e}_A\, X_b^A, \qquad X_a^A = \partial_a \chi^A$$

where (2.1)–(2.3) and (3.7) were taken into account. The *intrinsic plastic strain tensor* $\mathbf{E}_p$ is given then by

$$\mathbf{E}_p = \tfrac{1}{2}(\mathbf{G} - \mathbf{g}_0) \tag{3.19}$$

Taking into account that (Marsden and Hughes, 1978)

$$\mathbf{D}_g = \tfrac{1}{2} L_\mathbf{v}\mathbf{g}$$

$$\dot{\mathbf{G}} = \chi_t^*(\mathcal{L}_\mathbf{v}\mathbf{g}), \qquad \mathcal{L}_v = \partial_t + L_\mathbf{v} \tag{3.20}$$

where $L_\mathbf{v}$ denotes the Lie derivative operator, we obtain

$$\dot{\mathbf{E}}_p = \tfrac{1}{2}\dot{\mathbf{G}} = \chi^*(\mathbf{D}_g - \mathbf{D}_p) \tag{3.21}$$

where $\mathbf{D}_p$ denotes the rate of plastic stretchings tensor defined by (3.4) and (3.5b). It follows from (3.5a), (3.6), (3.16), and (3.21) that

$$\chi^*\mathbf{D}_g = \tfrac{1}{2}(\dot{\mathbf{G}} - \dot{\mathbf{g}}) \tag{3.22}$$

and

$$\operatorname{div}_g \mathbf{v} = \dot{J}_p/J_p \qquad \text{iff} \quad \operatorname{tr}\mathbf{D}_g = \operatorname{tr}\mathbf{D}_p \tag{3.23}$$

The relation (3.23) characterizes the case when the existence of a divergenceless dislocation flow velocity, i.e., the condition

$$\operatorname{tr}\mathbf{D}_g = 0 \tag{3.24}$$

is equivalent to the preservation of the body material volume in a rate-sensitive plastic regime. It is an intrinsic counterpart of the incompressibility condition in the theory of perfectly plastic materials.

If the material Riemannian space $\mathcal{B}_g$ is equidistant and the intrinsic metric tensor $\mathbf{g}$ takes, in a convective Lagrange coordinate system defined by (3.11), its canonical form (2.4) and (2.5), then the components L_{AB} defined by (3.9) and (3.11)–(3.14) take the following form:

$$L_{B3} = \nabla_B^g v_3 = 0 \tag{3.25a}$$

$$L_{3\alpha} = \nabla_3^g v_\alpha = \partial_3 v_\alpha + \kappa_t' v_\alpha \tag{3.25b}$$

$$L_{\beta\alpha} = \nabla^g_\beta v_\alpha = \partial_\beta v_\alpha - \Gamma^\kappa_{\beta\alpha}[\mathbf{a}]v_\kappa = \Psi_t \nabla^a_\beta \bar{v}_\alpha \tag{3.25c}$$

where $\nabla^a = (\Gamma^\kappa_{\alpha\beta}[\mathbf{a}])$ denotes the Levi-Civita covariant derivative corresponding to the metric tensor $\mathbf{a}(X^\kappa, t) = \mathbf{a}_t(X^\kappa)$ of a general (time-dependent) 2-dimensional Riemannian space [see (2.4)], the form of the Christoffel symbols $\Gamma^C_{AB}[\mathbf{g}]$ in actual coordinates of (2.4) was taken into account [Trzęsowski, 1998, (4.11)], and we denoted

$$v_\alpha = g_{\alpha\beta}v^\beta = \Psi_t \bar{v}_\alpha, \qquad \bar{v}_\alpha = a_{\alpha\beta}v^\beta \tag{3.26}$$

So, according to (3.15),

$$D_{33} = 0 \tag{3.27a}$$

$$D_{3\alpha} = \tfrac{1}{2}(\partial_3 v_\alpha + \kappa'_t v_\alpha) = D_{\alpha 3} \tag{3.27b}$$

$$D_{\alpha\beta} = \Psi_t d_{\alpha\beta}, \qquad d_{\alpha\beta} = \tfrac{1}{2}(\nabla^\alpha_\beta \bar{v}_\alpha + \nabla^a_\alpha \bar{v}_\beta) \tag{3.27c}$$

It follows from (2.15) and (3.27b) that

$$2D_{3\alpha} = \partial_3 v_\alpha + \tfrac{1}{2} H_t(X^3)v_\alpha \tag{3.28}$$

Equations (3.12) and (3.27c) suggest we consider the dislocation flow velocity as a field defining infinitesimal deformations $\boldsymbol{\eta}$ of surfaces $\Sigma_{c,t} = (\Sigma_c, \mathbf{a}_{c,t})$, where $\mathbf{a}_{c,t}$ is given by (2.13), that is (Hineva, 1984),

$$\boldsymbol{\eta} = \mathbf{w} + \eta\mathbf{n}, \qquad \mathbf{w} = t_d\mathbf{v}, \qquad [\mathbf{w}] = [1], \quad [\eta] = \text{cm}, \quad [t_d] = \text{sec} \tag{3.29}$$

where t_d is a constant, η is a scalar, and $\mathbf{n} = \mathbf{E}_3$ is the $\mathbf{g}$-unit normal to the surface. For example, $\boldsymbol{\eta}$ is an infinitesimal conformal deformation if (Hineva, 1984)

$$t_d D_{\alpha\beta} = t_d \Psi_t(c) d_{\alpha\beta} = \eta b[c, t]_{\alpha\beta} + \lambda a[c, t]_{\alpha\beta} \tag{3.30}$$

where λ, $[\lambda] = [1]$, is a scalar, and $b[c, t]_{\alpha\beta}$ are components of the second fundamental form $\mathbf{b}_{c,t}$ of the umbilical surface $\Sigma_{c,t}$ with the first fundamental form $\mathbf{a}_{c,t}$. Since for umbilical surfaces (Trzęsowski, 1998)

$$\mathbf{b}_{c,t} = \tfrac{1}{2} H_t(c)\mathbf{a}_{c,t} \tag{3.31}$$

the condition (3.30) takes the form [see (2.13) and (2.14b)]

$$t_d d_{\alpha\beta} = [\tfrac{1}{2} H_t(c)\eta + \lambda]a_{\alpha\beta} \tag{3.32}$$

Moreover, since in a coordinate system of (2.4), we have (Trzęsowski, 1998)

$$\Gamma^\kappa_{\alpha\beta}[\mathbf{g}] = \Gamma^\kappa_{\alpha\beta}[\mathbf{a}], \qquad \Gamma^3_{\alpha 3}[\mathbf{g}] = 0 \tag{3.33}$$

we obtain, taking into account (2.4), (2.13), (2.14b), (3.12), (3.16), and (3.17), that

$$\begin{aligned} \operatorname{div}_g \mathbf{v}_{|X^3=0} &= \operatorname{div}_a \mathbf{v}_0 \\ \mathbf{v}_0 = \mathbf{v}_{|X^3=0} &= v^\alpha(X^\kappa, 0, t)\partial_\alpha \\ \operatorname{tr}\mathbf{D}_g = g^{AB}D_{AB} &= a^{\alpha\beta}d_{\alpha\beta} \end{aligned} \tag{3.34}$$

It follows from (3.16), (3.30), (3.32), and (3.34) that along the surface $\Sigma_{0,t} = (\Sigma_0, \mathbf{a}_t) \subset \mathcal{B}_t$, the equation

$$\nabla^a_\alpha u_\beta + \nabla^a_\beta u_\alpha = (\operatorname{div}_a \mathbf{u}) a_{\alpha\beta}, \qquad \mathbf{u} = t_d \mathbf{v}_0 = u^\alpha(X^\kappa, t)\partial_\alpha \tag{3.35}$$

stating that the vector field $\mathbf{u}$ tangent to $\Sigma_{0,t}$ defines, at each instant $t \in I$, an infinitesimal conformal transformation of $\Sigma_{0,t}$, holds. If the condition (3.24) is fulfilled, then (3.35) reduces to the definition of $\mathbf{u}$ as a Killing vector field for the metric $\mathbf{a}_t$:

$$\nabla^a_\alpha u_\beta + \nabla^a_\beta u_\alpha = 0 \tag{3.36}$$

It is known (Eisenhart, 1964) that the two-dimensional manifold $\Sigma_{0,t} = (\Sigma_0, \mathbf{a}_t)$ has a constant scalar curvature $K_a(t)$ iff the isometry group G of the manifold has its maximal order 3. In this case the metric tensor $\mathbf{a}_t$ is reducible to

$$\begin{aligned} \mathbf{a}_t &= a_{\alpha\beta}\, dx^\alpha \otimes dx^\beta \\ a_{\alpha\beta} &= [1 + K_a(t) r^2/4]^{-2} \delta_{\alpha\beta}, \qquad r^2 = \delta_{\alpha\beta} x^\alpha x^\beta \end{aligned} \tag{3.37}$$

and generators of the group G are given by (Ikeda and Nishino, 1973)

$$\begin{aligned} &\mathbf{u}_a = \underset{a}{u}^\alpha \partial_\alpha, \qquad \partial_\alpha = \partial/\partial x^\alpha, \qquad \alpha = 1, 2, \qquad a = 1, 2, 3 \\ &\underset{\kappa}{u}^\alpha = [1 - K_a(t) r^2/4]\delta^\alpha_\kappa + \tfrac{1}{2} K_a(t) x^\alpha x_\kappa \\ &\underset{3}{u}^1 = x^2, \qquad \underset{3}{u}^2 = -x^1, \qquad x_\kappa = \delta_{\kappa\alpha} x^\alpha, \qquad \alpha, \kappa = 1, 2 \end{aligned} \tag{3.38}$$

Thus, the group G is locally isomorphic to $SO(3)$ $[K_a(t) > 0]$, $SO(2, 1)$ $[K_a(t) < 0]$, or $E(2)$ $[K_a(t) = 0]$, which denote the 3-dimensional rotation group, the 3-dimensional Lorentz group, and the 2-dimensional Euclidean group, respectively (Ikeda and Nishino, 1973).

Let us consider a Bravais moving frame with the intrinsic metric tensor $\mathbf{g}$ defined, in a convective Lagrange coordinate system of the form (3.11), by the conditions (2.4) and (2.5). If the Bravais moving frame admits the existence of a local glide system ($\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$) such that $\mathbf{n}$ is the unit normal to the surfaces $\Sigma_{c,t} = (\Sigma_c, \mathbf{a}_{c,t})$ [see (2.6) and (2.7)], then the surfaces $\Sigma_{0,t}$ can be considered as umbilical *glide surfaces* of a constant mean curvature $H_a(t)$ being virtual surfaces of the glide motion of edge dislocation lines defined by the local glide system. The Killing vector fields for the metrics $\mathbf{a}_t$, $t \in I$, define a class of dislocation flow velocities of the form (3.12) acting on the glide surfaces as their infinitesimal motions. In the *single glide* case (Section 1) planes originally parallel and normal to a lattice direction pass into glide surfaces without local stretchings (Bilby *et al.*, 1958). Consequently, such glide surfaces, say $\Sigma_{0,t}$ surfaces, ought to have vanishing scalar curvature $[K_a(t) = 0]$ and ought to be normal to a local lattice direction, say the $\mathbf{n} = \mathbf{E}_3$ direction. Thus the 2-dimensional Euclidean group $E(2)$ acts transitively on the single glide surfaces. This means that such a glide surface admits as its motion, in the small at least, the deformation of Euclidean reference lattice characterizing the influence of the glide motion on this lattice (Bilby *et al.*, 1958). For example, if the Bravais moving frame is defined by (2.8) and by the condition $[\mathbf{E}_\alpha, \mathbf{E}_\beta] = 0$, $\alpha, \beta = 1, 2$ (Trzęsowski, 1998, Section 5), then the local glide system $(\mathbf{l}, \mathbf{m}, \mathbf{n})$ defined by (2.10)–(2.12) describes such a physical situation. In this case the material equidistant space $\mathcal{B}_g$ has a constant negative scalar curvature (Trzęsowski, 1998). Note that the single glide surfaces embedded in a Euclidean space are called developable (Bilby *et al.*, 1958).

Let $\Sigma_{0,t}$ be the above defined umbilical glide surface, but of a constant negative scalar curvature $[K_a(t) < 0]$. For example, the Bravais moving frame defined by (2.8) admits this case. Since a 3-dimensional particular Lorentz transformation can be considered as a deformation of Euclidean plane changing a square into a rhomb (Trzęsowski, 1986) (so-called pure shear), this case may be interpreted as the one admitting locally the occurrence of a deformation process such that after passing through the transformation phase locally the crystal lattice regains its original structure, whereas globally one has a defect. This deformation results in macroscopic plastic deformation of a crystal (Rogula, 1975). The remaining 3-dimensional Lorentz transformations can be identified with 2-dimensional Euclidean rotations (see the simple glide case) or their composition with pure shearing.

The considered glide surfaces of a constant scalar curvature $K_a(t)$ are generalizations of a plane or sphere in a Euclidean 3-space. If $K_a(t) \leq 0$, then the surface can be locally considered as a plane [$K_a(t) = 0$—*parabolic*, i.e., locally Euclidean glide surfaces] or as a half-plane [$K_a(t) < 0$—*hyperbolic* glide surfaces]. The class of *elliptic* glide surfaces $[K_a(t) > 0]$ contains surfaces topologically (and even isometrically) equivalent to a Euclidean 2-sphere. In such a case we may expect that one part of the crystal is displaced (at least locally) by the action of 3-dimensional Euclidean rotations. It occurs in an elementary act of plasticity connected with the phenomenon of crystal fragmentation in the plastic yielding process and is called *rotational plasticity* (Panin *et al.*, 1985).

4. OROWAN-TYPE FORMULAS

Let (**l**, **m**, **n**) be a local glide system defining a foliation of the material Riemannian space $\mathcal{B}_g$ by glide surfaces normal to the **n** direction (Trzęsowski, 1998). Let **v** be a dislocation flow velocity with the vanishing climb component [i.e., $v_{(n)} = 0$ in (3.10)] and $\mathbf{D}_g$ the corresponding intrinsic rate of stretchings tensor defined by (3.13)–(3.15). The tensor field $\mathbf{D}_g$ is assumed to be constrained by the following counterparts of kinematic conditions considered in the theory of perfect plasticity (Gambin and Rychlewski, 1991): the "incompressibility" condition (3.24) and the condition of the existence of a family of instantaneously inextensible planes. The last condition can be formulated (in our notation) as follows: if **u** is a vector field such that

$$u_{(n)} = \mathbf{u} \cdot \mathbf{n} = 0, \qquad \text{i.e.,} \quad \mathbf{u} = u_{(l)}\mathbf{l} + u_{(m)}\mathbf{m} \tag{4.1}$$

where $\mathbf{u} \cdot \mathbf{v} = \mathbf{ugv}$, then

$$\mathbf{u}\mathbf{D}_g\mathbf{u} = 0 \tag{4.2}$$

The tensor field $\mathbf{D}_g$ has then, with respect to the local glide system, the following representation:

$$\mathbf{D}_g = \mathbf{g}\mathbf{D}\mathbf{g}^T \tag{4.3a}$$

$$\mathbf{D} = \frac{\dot{\gamma}}{2}(\mathbf{K} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{K}) \tag{4.3b}$$

where

$$\mathbf{K} = \mathbf{m} + \delta_D\mathbf{l} = K_g\mathbf{k}, \qquad \mathbf{k} \cdot \mathbf{k} = 1, \qquad \mathbf{k} \cdot \mathbf{n} = 0$$
$$K_g = \|\mathbf{K}\|_g = (1 + \delta_D^2)^{1/2}, \qquad \delta_D = D_{ln}/D_{mn}$$
$$D_{ln} = \mathbf{lDn}, \qquad D_{mn} = \mathbf{mDn} = \dot{\gamma}/2 \tag{4.4}$$

and $\dot{\gamma}$ denotes the rate of the (inelastic) shear in the **k** direction due to the dislocation flow. If

$$\mathbf{n} = \partial_3$$
$$\mathbf{K} = K^\alpha\partial_\alpha, \qquad K_\alpha = g_{\alpha\beta}K^\beta, \qquad \alpha, \beta = 1, 2 \tag{4.5}$$

where $X = (X^A) = (X^\kappa, X^3)$ is a Lagrange coordinate system on $\mathcal{B}_g$, then it follows from (4.3)–(4.5) that the components D_{3a} of $\mathbf{D}_g$ take the form

$$D_{3\alpha} = \frac{\dot{\gamma}}{2} K_\alpha, \qquad K_\alpha = m_\alpha + \delta_D l_\alpha = K_g k_\alpha \tag{4.6}$$

The pair (**k, n**) defines a local slip system with a *resulting slip* in the **k** direction caused by a glide motion of edge dislocation lines (being lines of the vector field **l**) with the slip direction **m** and by an additional slip in the **l** direction tangent to these lines.

If the material Riemannian space is equidistant and the canonical form (2.4) and (2.5) of its metric tensor **g** is attained in actual coordinates defined by (3.11), then it follows from (3.28) and (4.6) that the resulting slip direction covers the dislocation flow velocity direction, that is,

$$\mathbf{v} = v_g \mathbf{k}, \qquad v_g > 0 \tag{4.7}$$

if

$$\partial_3 v_\alpha = 0 \tag{4.8}$$

Then

$$v_g = 2K_g \frac{\dot{\gamma}}{H_t} \tag{4.9a}$$

$$\operatorname{sgn} \dot{\gamma} = \operatorname{sgn} H_t \tag{4.9b}$$

and, according to (4.8), the actual coordinates can be taken in the form (2.9). Consequently, if the distribution of dislocations is defined by (2.8), the glide system is defined by (2.10) and (2.11), and

$$\dot{\gamma} > 0 \tag{4.10}$$

then we obtain, according to (2.16) and (4.9), the following generalization of the *Orowan relation* (Trzęsowski, 1998, Section 1):

$$\dot{\gamma} = K_g^{-1} \rho b_g v_g \tag{4.11}$$

where b_g is the modulus of the local Burgers vector **b** defined by (2.11). Since $K_g = 1$ iff local slips are caused by a *glide motion* only

$$\mathbf{k} = \mathbf{m} \tag{4.12}$$

the scalar K_g of (4.11) is a *directional coefficient* (Perzyna, 1978). In the case (4.12), equation (4.11) reduces to the well-known form of the Orowan relation:

$$\dot{\gamma} = \rho b_g v_g \tag{4.13}$$

The tensor **D** defined by (4.3b) takes then the form

$$\mathbf{D} = \dot{\gamma}\mathbf{M}, \qquad \mathbf{M} = \tfrac{1}{2}(\mathbf{m} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{m}) \tag{4.14}$$

If, for the same local glide system, a *double cross-slip* (Section 1) in the $\mathbf{m}$ direction is considered, then $\partial_3 v_\alpha \neq 0$ and, according to (3.28) and (4.6), we have

$$K_g \dot{\gamma} k_\alpha = \partial_3 v_\alpha + \tfrac{1}{2} H_t v_\alpha \tag{4.15}$$

If additionally

$$\mathbf{K}(X, t) = K_g(X^3, t)\mathbf{k}(X^\kappa, t) \tag{4.16}$$

and the conditions (4.7) and (4.10) are fulfilled, then we obtain the following extension of the generalized Orowan relation (4.11):

$$\dot{\gamma} = K_g^{-1}(\rho b_g v_g + \partial_{\mathbf{n}} v_g) \tag{4.17}$$

where (2.6) and (2.16) were taken into account. $K_g = 1$ means then that local slips are caused by a double cross-slip process only:

$$\dot{\gamma} = \rho b_g v_g + \partial_{\mathbf{n}} v_g \tag{4.18}$$

We *conclude* that the equidistant property of the Riemannian material space $\mathcal{B}_g$ is consistent with the glide motion of dislocations as well as with the double cross-slip process. It is also a property of the material space consistent with an ability of dislocations to organize themselves in periodic layers (e.g., in the form of slip bands; see Sections 1 and 5). Moreover, it appears that the equidistant property of $\mathcal{B}_g$ admits the existence of distributions of dislocations [defined by (2.8)] and the existence of dislocation flows [defined by (2.9) or (3.11)] for which the Orowan-type kinematic formulas are valid. This suggests that this particular case of the material geometry is of essential physical importance.

5. FINAL REMARKS

Let $(\mathbf{m}, \mathbf{n})$ denotes the local slip system of the local glide system defined by (2.10) and (2.11) and corresponding to the distribution of dislocations defined by (2.8). Let us consider a dislocation flow defined by (2.9) with the dislocation flow velocity (3.9) parallel to the slip direction $\mathbf{m}$ [see (4.7)–(4.13)]. The intrinsic rate of stretchings tensor $\mathbf{D}_g$ is given then by (4.3a) and (4.14). If $\mathbf{T}$ is a symmetric stress tensor defined in actual configurations $\chi_t(U) \subset \mathcal{B}$, $t \in I$, of domains $U \subset \mathcal{B}$, then the scalar

$$T = \mathbf{TM} = \mathbf{mTn} \tag{5.1}$$

is the so-called *resolved shear stress* in a local slip plane normal to the $\mathbf{n}$ direction and containing the slip direction $\mathbf{m}$. There are various dislocation dynamic descriptions treating T as a driving stress of moving dislocations.

For example, it has been experimentally determined that at low temperatures [when the climb is negligible (Ghoniem and Amadeo, 1990); see (3.10) with $v_{(n)} = 0$] the relationship between the (mean) glide velocity v_g and stresses has the form (Ghoniem and Amadeo, 1990; Perzyna 1978; Yang and Lee, 1993)

$$v_g = v_0 \left(\frac{T}{T_0} \right)^n \tag{5.2}$$

where v_0 is the shear wave velocity, and T is the effective resolved stress. T_0 as well as n may be, in general, functions dependent on the temperature and permanent strains; in particular cases they may be treated as material constants. It is not assumed in this case that a critical value of the stresses is needed for the activation of the dislocation motion (and thereby to create conditions for the appearance of plastic deformation) (Perzyna, 1978).

It is considered, in order to introduce the notion of critical stresses, an equivalent definition of T providing the alternative physical significance of the resolved shear stress (called also then the Schmid resolved shear stress) (Yang and Lee, 1993). Namely, since the intrinsic rate of stretchings tensor $\mathbf{D}_g$ is a measure of the rate of stretchings during the plastic deformation due to the glide motion of dislocations (Section 4), the dissipated power per unit mass can be introduced by

$$W = \mathbf{T} \cdot \mathbf{D}_g = T\dot{\gamma} \tag{5.3}$$

where T is defined by (5.1) and $\mathbf{D}_g$ by (4.3a) and (4.14). It states that the resolved shear stress bears the physical significance of the plastic work conjugate of shear rate $\dot{\gamma}$ in the local slip system ($\mathbf{m}$, $\mathbf{n}$). Moreover, the condition of nonnegativeness of dissipation should be fulfilled:

$$T\dot{\gamma} \geq 0 \tag{5.4}$$

The *Schmid yield criterion* refers to cases where the variations of shear rate $\dot{\gamma}$ only depend on the corresponding shear stress, and states that

$$\begin{aligned} \dot{\gamma} &= 0 \qquad \text{for} \quad 0 \leq T < T_c \\ \dot{\gamma} &\geq 0 \qquad \text{for} \quad T \geq T_c > 0 \end{aligned} \tag{5.5}$$

where T_c is a *critical resolved shear stress* and the conditions (4.10) and (5.4) were taken into account. The slip system in which T reaches a critical value T_c is usually termed the *critical slip system* and provides the so-called *yield surface* in stress space defined by the condition

$$\mathbf{mTn} = T_c \tag{5.6}$$

For the *rate-dependent materials* in the regime of glide with normal speed (i.e., $v_g < c_d$, where c_d is a quantity of the order of the elastic shear wave speed), the shear rate $\dot{\gamma}$ of the Schmid yield criterion is usually characterized by the following macroscopic counterpart of the power low (5.2) (Yang and Lee, 1993; Batra and Zhu, 1995):

$$\dot{\gamma} = \begin{cases} \dot{\gamma}_0\left(\dfrac{T}{T_c}\right)^{1/m} & \text{for} \quad T \geq T_c \\ 0 & \text{for} \quad T < T_c \end{cases} \tag{5.7}$$

where $\dot{\gamma}_0$ is a characteristic reference strain rate such that if the crystal is deformed with $\dot{\gamma} = \dot{\gamma}_0$ [in our case the deformation is a material flow χ: $\mathcal{B}_g \times I \to \mathcal{B}_g$ and $\dot{\gamma}$ is that of (4.14)], $T = T_c$ is a critical resolved shear stress required to cause a plastic deformation in the (local) slip system (**m**, **n**).

The derived Orowan formula (4.13) is a bridge between micro- and macromechanics [i.e. between the relations (5.2) and (5.7)]. A second relation between micro- and macromechanics [see (2.16), (4.9b), and (4.10)]

$$\rho b_g = \tfrac{1}{2} H \tag{5.8}$$

describes the influence of microscopic quantities ρ and b_g (the scalar density of dislocations and the modulus of the local Burgers vector, respectively) on the mean curvature H of umbilical slip surfaces [nonplanar due to the influence of secondary point defects (Trzęsowski, 1998)]. It follows from (4.13) and (5.8) that

$$\dot{\gamma} = \tfrac{1}{2} H v_g \tag{5.9}$$

where v_g is the modulus of the velocity of edge dislocations gliding over a slip surface of the mean curvature H. The relation (5.9) concerns a particular distribution of dislocations for which their mobility produces a plastic shearing due to the existence of secondary point defects created by this distribution of dislocations. The dislocation fluid case (Trzęsowski, 1998) provides an example for which, in general (Trzęsowski, 1998, Section 5), the relation (5.8) is not valid. Note that although the instantaneous slip surfaces $\Sigma_{c,t} \subset \mathcal{B}_t$ (Section 2) can be locally isometrically embedded in the 3-dimensional Euclidean configurational space of the body (Friedman, 1965), the mean curvature H is not preserved under this embedding [since H is a relative geometric quantity (Trzęsowski, 1998, Section 4, (4.17))]. Consequently, the mean curvature H may be treated as a material parameter. For example, it follows from (5.2) and (5.9) that the shear rate $\dot{\gamma}$ due to the driving stress of moving dislocations has the form resembling that, (5.7), of the Schmid yield criterion:

$$\dot{\gamma} = \dot{\gamma}_0\left(\frac{T}{T_0}\right)^n, \qquad \dot{\gamma}_0 = \tfrac{1}{2} H v_0 > 0 \tag{5.10}$$

where (in general) $\dot{\gamma}_0 = \dot{\gamma}_0\,(X^3, t)$ is a characteristic strain rate; if $\mathcal{B}_g$ is an Einstein space [and thus $\mathcal{B}_g$ is conformally flat (Trzęsowski, 1998, Sections 4 and 5], then $\dot{\gamma}_0 = \dot{\gamma}_0\,(t)$. Since (for $T \neq 0$) $\dot{\gamma} = 0$ iff $\dot{\gamma}_0 = 0$, we may consider (5.10) as a model of the phenomenon of localized plastic shearing, say on the inside of a layer of the thickness $2h$ consisting of equidistant glide surfaces (see Sections 1 and 2), assuming that

$$
\begin{aligned}
H(X^3, t) > 0 \qquad &\text{for} \quad |X^3| < h \\
H(X^3, t) = 0 \qquad &\text{for} \quad |X^3| \geq h
\end{aligned}
\tag{5.11}
$$

Then, the considered mobile dislocation lines are, according to (5.8), absent on the outside of the layer. Assuming that $T_0 = T_c$ is a critical resolved shear stress, we can extend (5.10) to the macroscopic formula (5.7) [considered on the inside of the layer in the case (5.11)].

We *conclude* that the proposed geometric model of the edge dislocation kinematics has, at least for some particular cases of distributions of dislocations admitting the relation (5.8), the Orowan-type mesoplastic character.

REFERENCES

Batra, R. G., and Zhu, Z. G. (1995). *Acta Mechanica,* **113**, 185.

Bilby, B. A., Bullough, R., Garder, L. R., and Smith, E. (1958). *Proceedings of the Royal Society A,* **244**, 538.

Eisenhart, P. E. (1964). *Riemannian Geometry,* Princeton University Press, Princeton, New Jersey.

Friedman, A. (1965). *Reviews of Modern Physics,* **37**, 201.

Gambin, W., and Rychlewski, J. (1991). *Archives of Mechanics,* **23**, 765.

Ghoniem, N. M., and Amadeo, R. J. (1990). In *Patterns, Defects and Material Instabilities,* D. W. Walgraef and N. M. Ghoniem, eds., Kluwer, Dordrecht, pp. 303–309.

Hineva, S. T. (1984). In *Differential Geometry,* W. Waliszewski, ed., PWN, Warsaw.

Hull, D., and Bacon, D. J. (1984). *Introduction to Dislocations*, Pergamon Press, Oxford.

Ikeda, M., and Nishino, Y. (1973). *Tensor,* **7**, 295.

Kröner, E. (1986). *ZAMM,* **66**, T284.

Marsden, J. E., and Hughes, T. J. R. (1978). In *Nonlinear Analysis and Mechanics*, Vol. 2, R. J. Knops, ed., Pitman, London, pp. 30–269.

Panin, W. E., Lihatchev, W. A., and Grinaev, J. W. (1985). *Structural Levels of Deformations in Solids*, Science, Novosibirsk, Russia [in Russian].

Perzyna, P. (1978). *Thermodynamics of Inelastic Materials*, PWN, Warsaw [in Polish].

Rogula, D. (1975). In *Trends and Applications of Pure Mathematics to Mechanics*, G. Fishera, ed., Pitman Publishing, London, pp. 311–331.

Trzęsowski, A. (1986). *International Journal of Theoretical Physics,* **26**, 311.

Trzęsowski, A. (1993). *Reports on Mathematical Physics,* **1**, 71.

Trzęsowski, A. (1998). *International Journal of Theoretical Physics*, this issue.

Yang, W., and Lee, W. B. (1993). *Mesoplasticity and Its Applications*, Springer-Verlag, Berlin.